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Dynamics of the pressure relaxation in a "depressurized" borehole☆

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ABSTRACT

A problem of the pressure relaxation in a borehole after its "depressurization" is considered. An integral equation describing the evolution of the pressure in the borehole is obtained. It is shown that, by a choice of the initial bulk gas content and the height of the Impermeable part of the borehole, it is possible to obtain that the half-period of the pressure relaxation in the borehole lies within limits which are convenient for the technical realization of "depressurization". Nomograms are constructed which enable one, when the values of the borehole parameters are known, to estimate the permeability of the surrounding porous medium using the half-period of the pressure relaxation.

In practice, hydrodynamic methods of investigating a stratum are mostly used to obtain information on the collector properties of the stratum. The most widely used method is to plot curves of the presurre relaxation at the end face of a borehole and of the changes in the output and pressure.[1,2] Several methods exist for determining the collector characteristics of a stratum using the curve of the pressure relaxation in the borehole, including the methods of arbitrary and characteristic points.[1]

Below, we propose that the half-period for the pressure relaxation in a "depressurized" borehole can be is used to estimate the collector characteristics of a stratum. We shall call the period of time during which the difference between the pressure in the borehole and the pressure in the porous medium hlaves in value the half-period for the pressure relaxation. A sharp decrease in the pressure in the borehole is called a "depressurization". It can be achieved, for example, by collapsing containers, containing gas at a lower pressure than the pressure in the stratum, which have been lowered into the borehole.

The pressure relaxation in a cavity surrounded by a porous and permeable rock has been investigated.[3,4] The problem of the pressure testing, of a liquid-filled cavity (a crack or a cylindrical or spherical void) By injecting of a certain amount of gas has been considered and the dependence of the relaxation time on the initial pressure drop has been studied for the case when the whole of the lateral surface of the borehole is permeable.[3] The evolution of the pressure in a cavity surrounded by a porous medium saturated with gas after an explosion has been investigated.[1]

1. Basic equations

Suppose, in the initial state ($t<0$), the pressure of a liquid in the whole of an unbounded porous stratum outside a borehole (Fig. 1) is constant and equal to p_0'. We will consider the relaxation of pressure p in a certain segment of the borehole between two impermeable end boundaries to the value p_0'. Suppose an initial pressure $p_0 (p_0 \le p_0')$ is established in the borehole at the instant $t=0$.

In order to describe the pressure relaxation, we will assume that the pressure is uniform within the borehole (we will neglect the hydrostatic pressure drop) and that there are no phase transitions (the mass of gas within the borehole remains unchanged). The isolated segment of the borehole of height h consists of a permeable part of height h^{op} and an impermeable part of height h^{cl}. We will assume that the permeable part penetrates the whole thickness of the stratum, and the wall of the remaining part of the borehole, that is, above the roof and below the base, is impermeable (the impermeable part). The ends of the isolated segment of the borehole, and the roof and the base of the stratum are also impermeable. By covering the end faces of the isolated segment of the borehole at different places, it is possible to control the height of the impermeable part, while the height of the permeable part is assumed to be considerably greater than its radius a.

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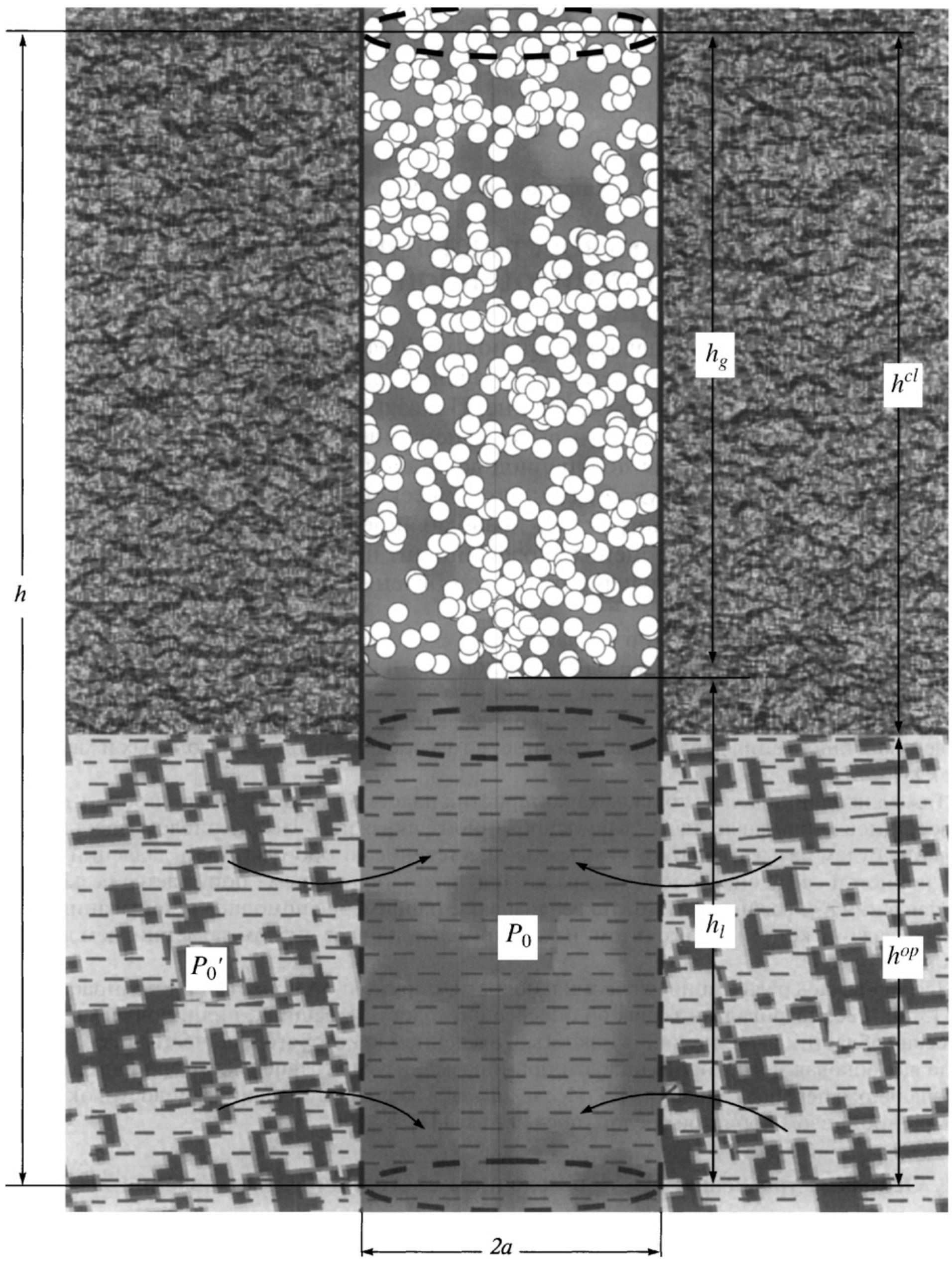

Fig. 1.

We will write the law for the change in the mass of the liquid m_l within the borehole in the form

$$\frac{dm_l}{dt} = S\rho_l \upsilon_a; \quad m_l = \pi a^2 h_l \rho_l, \quad S = 2\pi a h^{\mathrm{op}}, \quad h = h^{\mathrm{op}} + h^{\mathrm{cl}} = h_g + h_l \tag{1.1}$$

where S is the area of the lateral surface of the borehole through which liquid seepage occurs, ρ_l is the liquid density, υ_a is the rate of seepage through the borehole walls, and h_g and h_l are the reduced heights of the parts of the borehole occupied by the gas and the liquid.

We will take the equation of state of the liquid in the borehole and the porous medium in the acoustic approximation

$$p = p_0 + C_l^2(\rho_l - \rho_{l0}) \tag{1.2}$$

where C_l is the velocity of sound in the liquid and ρ_{l0} is the initial liquid density of the liquid. We will assume that the gas is calorifically perfect and its behaviour obeys the polytropic law

$$h_g = h_{g0}(p_0/p)^{1/\gamma} \tag{1.3}$$

where γ is the polytropic exponent and h_{g0} is the reduced height of the part of the borehole occupied by the gas phase at the initial instant. We will introduce the volume fraction of the gas α_g in the borehole as α_g=h_g/h.

We will use Darcy's law

$$\upsilon' = -\frac{k}{\mu_l}\frac{\partial p'}{\partial r}, \quad a < r < \infty, \tag{1.4}$$

to describe the inflow of the liquid into the well, where υ' and p' are the seepage rate and the liquid pressure of the liquid in the porous medium, k is the permeability of the porous medium and μ_l is the dynamic viscosity of the liquid.

The plane-radial seepage of the liquid in the porous medium outside the borehole is described by the piezoconduction equation[5]

$$\frac{\partial p'}{\partial t} = \chi\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p'}{\partial r}\right), \quad a < r < \infty, \quad \chi = \frac{k\rho_{l0}C_l^2}{m\mu_l} \tag{1.5}$$

where m is the porosity.

The assumptions mentioned above enable us to write the initial and boundary conditions for Eq. (1.5) in the form

$$t = 0, \quad r > a: p' = p_0' \tag{1.6}$$

$$t > 0, \quad r = a: p' = p(t); \quad t > 0, \quad r \to \infty: p' = p_0' \tag{1.7}$$

where $p(t)$ is the current unknown pressure in the borehole. Since the liquid density depends only slightly on the pressure ($\rho_l - \rho_{l0} \ll \rho_l$), we shall neglect the change in the liquid density on the right-hand side of Eq. (1.1), putting $\rho_l = \rho_{l0}$

From Eq. (1.1), we obtain

$$\alpha_{g0}\left(\left(\frac{p_0}{p}\right)^{1/\gamma} - 1\right) - \frac{p - p_0}{\rho_{l0}C_l^2}\left(1 - \alpha_{g0}\left(\frac{p_0}{p}\right)^{1/\gamma}\right) = -\frac{2\eta k}{a\mu_l}\int_0^t\left(\frac{\partial p'}{\partial r}\right)_a dt', \quad \eta = \frac{h^{\mathrm{op}}}{h} \tag{1.8}$$

We determine the liquid pressure in the porous medium outside the borehole using piezoconduction Eq. (1.5). The solution of this equation, obtained using the Duhamel principle with initial and boundary conditions (1.6) and (1.7), has the form

$$p' - p_0' = \int_0^t \frac{\partial U(r, t - t')}{\partial t}\left(p(t') - p_0'\right)dt'$$

$$U(r,t) = 1 + \frac{2}{\pi}\int_0^\infty \exp\left(-\frac{z^2 t}{t_a}\right)\frac{J_0\left(\frac{zr}{a}\right)Y_0(z) - J_0(z)Y_0\left(\frac{zr}{a}\right)}{J_0^2(z) + Y_0^2(z)}\frac{dz}{z}, \quad t_a = \frac{a^2}{\chi} \tag{1.9}$$

where $J_0(z)$ and $Y_0(z)$ are zero order Bessel and Neumann functions. The function $U(r,t)$ is the solution of Eq. (1.5) with the initial and boundary conditions [6]

$$r = a: p' = 1; \quad r \to \infty: p' = 0; \quad t = 0: p' = 0$$

After substituting expression (1.9) into equality (1.8) and some reduction, we obtain the following non-linear equation describing the evolution of the pressure in the "depressurized" borehole

$$\alpha_{g0}\left(\left(\frac{p_0}{p}\right)^{1/\gamma} - 1\right) - \frac{p - p_0}{\rho_{l0}C_l^2}\left(1 - \alpha_{g0}\left(\frac{p_0}{p}\right)^{1/\gamma}\right) = \frac{k\eta}{a^2\mu_l}\int_0^t \varphi\left(\frac{t - t'}{t_a}\right)\left(p(t') - p_0'\right)dt'$$

$$\varphi(x) = \frac{8}{\pi^2}\int_0^\infty \frac{\exp(-xz^2)}{J_0^2(z) + Y_0^2(z)}\frac{dz}{z} \tag{1.10}$$

Equation (1.10) was obtained and analysed earlier [3] for the case when $h^{\mathrm{cl}} = 0$.

In the case of a weak initial pressure drop ($\Delta p_0 = p_0' - p_0 \ll p_0$), from Eq. (1.10) we have

$$\Delta P = -\frac{1}{\beta t_a}\int_0^t \varphi\left(\frac{t - t'}{t_a}\right)(\Delta P(t') - 1)dt' \tag{1.11}$$

Here,

$$\Delta P = \frac{\Delta p}{\Delta p_0}, \quad \Delta p = p - p_0, \quad \beta = \frac{\alpha_{g0} + \gamma\alpha_C(1 - \alpha_{g0})}{\eta\gamma m\alpha_C}, \quad \alpha_C = \frac{p_0}{\rho_{l0}C_l^2}$$

The kernel of integral equation (1.11) has the following expansions for small and large values of the argument [3]

$$\varphi(x) = \frac{2}{\sqrt{\pi x}} + 1 - \frac{1}{2}\sqrt{\frac{x}{\pi}} + \frac{x}{4} + \cdots, \quad x \ll 1 \tag{1.12}$$

$$\varphi(x) = \frac{4}{\ln(4x/\Gamma)}, \quad \Gamma = \exp(2C), \quad x \gg 1 \tag{1.13}$$

where C is Euler's constant.

The first two terms of expansion (1.12) can be used [4] for the kernel in the time interval $0 < t \leq 10t_a$. In this case, the solution of integral equation (1.1) can be written in the form

$$\Delta P = 1 - \mathrm{Re}\left[\frac{1}{\beta_+ - \beta_-}\left[\beta_+\exp\left(\beta_+^2\tau\right)\Phi\left(\beta_+\sqrt{\tau}\right) - \beta_-\exp\left(\beta_-^2\tau\right)\Phi\left(\beta_-\sqrt{\tau}\right)\right]\right]$$

$$\Phi(\xi) = 1 - \frac{2}{\sqrt{\pi}}\int_0^\xi \exp(-\lambda^2)d\lambda, \quad \tau = \frac{t}{\tilde{t}}, \quad \tilde{t} = \beta^2 t_a, \quad \beta_\pm = 1 \pm \sqrt{1-\beta} \tag{1.14}$$

Solution (1.13) describes the dynamics of the initial stage of the pressure relaxation in the borehole. If the relaxation half-period t_p satisfies the condition $t_p \leq 10t_a$, then this solution can be used during the whole half-period.

A somewhat different approximate analytical solution can be constructed for the concluding stage of the pressure relaxation. We will assume that, at this stage starting from a certain instant t_*, the current pressure drop in the borehole and the inflow of liquid through its wall are related by the equality

$$\upsilon_a = \frac{2(p - p_0')k\eta}{\mu_l a \ln(4t/(\Gamma t_a))} \tag{1.15}$$

which follows from the known self-similar solution [7] when $t \gg t_a$. Then, substituting expression (1.15) into Eq. (1.1), we obtain the solution for the final stage of the pressure relaxation in the form of the following quadrature

$$\int_{p_*}^{p} \frac{\left((1-\alpha_g)/C_l^2 + \rho_l\alpha_g/(\gamma p)\right)dp}{p - p_0'} = -\frac{4k\rho_{l0}\eta}{a^2\mu_l}\int_{t_*}^{t}\frac{dt'}{\ln(4t'/(\Gamma t_a))} \tag{1.16}$$

Here, p_* is the pressure in the borehole at the instant $t = t_*$, and α_g and ρ_l are functions of the pressure which are defined by expressions (1.2) and (1.3). In the case of a weak initial pressure drop, it follows from equality (1.16) that

$$p = p_0' + \Delta p_* \exp\left(-\frac{4}{\beta t_a}\int_{t_*}^{t}\frac{dt'}{\ln(4t'/(\Gamma t_a))}\right), \quad \Delta p_* = p_* - p_0' \tag{1.17}$$

2. Dynamics of pressure relaxation in the borehole

The curves for the relaxation of the dimensionless pressure ΔP in the borehole after its "depressurization" for different values of the height of the impermeable part h^{cl}, obtained by numerical solution of Eq. (1.10), are shown on the left-hand side of Fig. 2. The values of the parameters of the borehole, the porous medium, the liquid and the gas were as follows:

$$a = 0.1\ \mathrm{m}, \quad m = 0.1, \quad k = 10^{-12}\mathrm{m}^2, \quad h^{\mathrm{op}} = 60\ \mathrm{m}, \quad p_0' = 10\ \mathrm{mPa}, \quad p_0 = 1\ \mathrm{mPa},$$

$$\rho_{l0} = 1000\ \mathrm{kg/m^3}, \quad C_l = 1.5\cdot 10^3\ \mathrm{m/s}, \quad \mu_l = 0.001\ \mathrm{Pa\cdot s}, \quad \gamma = 1.4, \quad \alpha_{g0} = 0.95$$

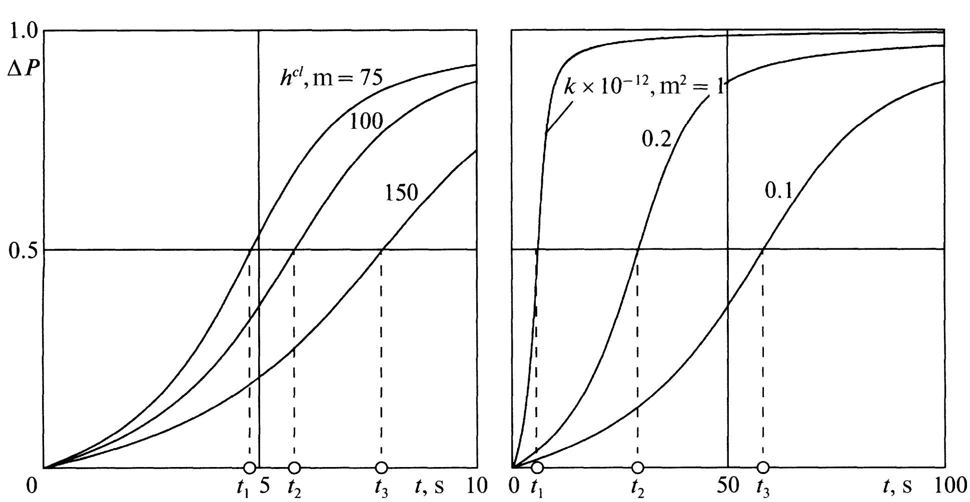

Fig. 2.

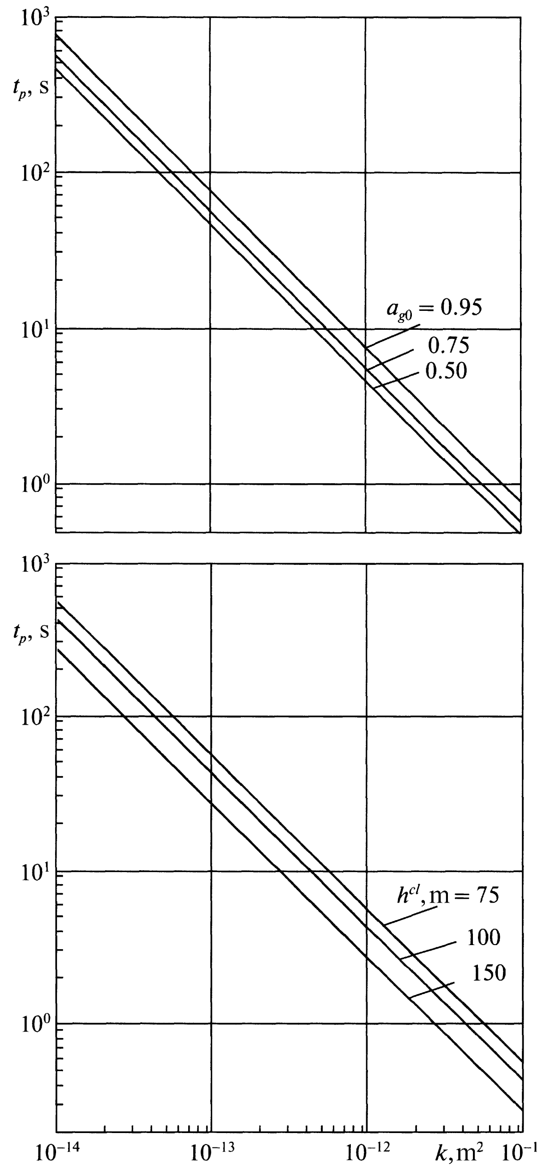

Fig. 3.

Unless otherwise stated, the same values were also used for the parameters of the borehole - porous medium system in the subsequent numerical examples. The points on the abscissa t_1, t_2 and t_3 are the half-periods for the pressure relaxation for the corresponding curves. It is clear that the half-period for the pressure relaxation depends on the height of the impermeable part of the borehole h^{cl}.

The curves for the pressure relaxation in a borehole for h^{cl} = 100 m and different values of the permeability k are shown on the right-hand side of Fig. 2. It is clear that the half-period for the pressure relaxation t_p decreases in inverse proportion to the permeability k.

The dependence of the dynamics of the pressure relaxation in a borehole on the values of the parameter α_{g0} and the ratio of the parameters h^{cl}/h^{op} was analysed in a similar manner and showed that the half-period for the pressure relaxation t_p increases as the values of these parameters increase.

The dependence of the time t_p on the permeability k is shown in Fig. 3 for $h^{cl} = 75$ and different initial gas contents α_{g0} (above)) and for $\alpha_{g0} = 0.75$ and different values of the height of the impermeable part of the borehole h^{cl} (below). The relations can be considered as nomograms for determining the value of the permeability k of the porous medium surrounding the borehole. For this purpose, the value of the half-period for the pressure relaxation t_p is determined using the experimental curve for the pressure relaxation in the "depressurized" borehole. Then, using the known values of α_{g0}=0.75 and h^{cl}, one or other line is chosen in the nomogram, on which the point corresponding to the time t_p is determined. The abscissa of this point then corresponds to the required value of the permeability k.

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